TOWARDS SPECTRAL DECOMPOSITIONS OF MULTICHANNEL COHERENT SIGNALS – CONSIDERATIONS OF CLIFFORD INTEGRAL TRANSFORMS OF POLARIMETRIC RADIO SIGNAL

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Modern digital radio systems enable operation of multiple mutually coherent channels (MMCC). The signals can be modelled as complex vector valued functions of Euclidian real space (f: $\mathbb{R}^n \to \mathbb{C}^m$) and they carry conceptually multidimensional (mD) information. Polarimetric radar and multiple antenna wireless communications (MIMO) are examples of MMC hardware realizations.

Polarization of electromagnetism is considered in interferometric radio astronomy and in optics, thus providing with traditional formalisms [1] of complex 2D signals. Pending for consistent spectral methods to analyse intrinsically 2D information, correlograms i.e. auto- and cross-correlation functions are used to interpret the signals in the 'mean sense'. 1D spectral decompositions (such as Fast Fourier Transforms, FFT) are widely used in engineering applications, and they can be reapplied to each channel of MMCC. However, the differential mD information continue to be inferred from averaged auto- and cross-correlation estimates. In operational polarimetric Doppler weather radar [2] (PDWR), new opportunities have emerged from multipurpose end uses [3] calling for more detailed decomposition of MMCC signals beyond the 'mean sense' estimators.

As extensions to FFT of real or complex valued 1D signals, various quaternion (QFT), Clifford (CFT) and geometric algebra (GAFT) transforms have been proposed for generalized spectral analyses of mD information. Orthogonal Plane Split (OPS) CFT [4] indicates how to carry over the favourable FFT properties, such as inverse, Plancherel-Parseval and convolution theorems. Steerable nature of OPS-CFT suggests for a continuum spectral decompositions to be available for examination of the signal feature space.

Here, the steps are clarified for composing PDWR signals as real quaternions ($\in Cl(0;2)$, isomorphic to SO(3) rotations). Subsequently, OPS-CFT is applied to simulated echo to recognize features of application interest. Examples of PDWR data are shown.

[1] J.P. Hamaker. J.D. Bregman and R.J. Sault, Understanding radio polarimetry. I. Mathematical foundations, Astron. and Astrophys. Suppl. Ser. **117**, 137-147 (1996).
[2] V.N. Bringi, and V. Chandrasekar, Polarimetric Doppler Weather Radar: Principles and Applications. Cambridge University Press, 636 pp. (2001).
[3] European Network for the Radar surveillance of Animal Movement, http://ow.ly/XJeNN; Am. Met. Soc.: A Course on Radar Aeroecology (2015), http://ow.ly/XJeV8.
[4] E. Hitzer, Two-Sided Clifford Fourier Transform with Two Square Roots of -1 in Cl(p;q), Adv. Appl. Cliff Algebras, **24**, 313-332, (2014), http://link.springer.com/article/10.1007%2Fs00006-014-0441-9