DERIVATION OF THE CLIMATE SENSITIVITY USING THE FREQUENCY RESPONSE OF THE CLIMATE SYSTEM

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The impulse and step responses of the climate system include the climate sensitivity. However, a problem is that the climate does not create changes, which we can use as an impulse or step response in order to derive the sensitivity. There are a few studies, where an eruption has been used as an impulse response. One very promising possibility is to use the frequency response (transfer function) of the climate, which is the Laplace transform of the impulse response. The frequency response corresponds to the impulse response in the frequency domain. So we move from the time domain to the frequency domain. The best measurable signal nature gives us all the time, is the annual temperature curve, which includes all the information about the response of the climate. The impulse response is $\exp(-t/\tau)/\tau$, where the response time $\tau = RC$, the product of the sensitivity and the heat capacity of the climate. If the impulse response were an infinitely narrow Dirac delta function we had, for a sinusoidal solar forcing, the temperature change $\Delta T = RA\cos(\omega t)$, where time starts in summer solstice, June 21 in the Northern and December 22 in the Southern Hemisphere, and A is the amplitude of the solar forcing. The angular frequency $\omega = 2\pi/year$. However, the non-zero response time causes lag and attenuation on the temperature change according to the equation

$$\Delta T = RA\cos\phi\cos(\omega t - \phi), \quad (1)$$

where the delay is given in the form of the phase shift $\phi = \arctan\omega\tau$. We have derived the same equation also as the convolution of impulse response and solar forcing in paper [1]. At the temperature maximum or minimum we have $\cos(\omega t - \phi) = \pm 1$ and hence

$$R = \frac{\Delta T}{A\cos\phi} \quad (2)$$

The temperature reaches maximum in the Northern Hemisphere with the phase shift $\phi_N = 39.5°$ and minimum in the Southern Hemisphere with the phase shift $\phi_S = 48.3°$ [2]. According to Fig. 2.7 in [3] the solar forcing $A_N = A_S = 235$ W/m^2 at the latitudes $60°$ and $-60°$, respectively. Figure 1.5 in [3] gives the corresponding temperature amplitudes $\Delta T_N = (33°C)/2$ and $\Delta T_S = (10°C)/2$. Substitution into Eq. (2) gives $R_N = 0.091°\mathrm{C}/(\mathrm{W/m^2})$ and $R_S = 0.032°\mathrm{C}/(\mathrm{W/m^2})$. The average of these is the global climate sensitivity $R = 0.0615°\mathrm{C}/(\mathrm{W/m^2})$. This value is in good agreement with the values derived in our papers [1] and [4].

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