SUPERFLUID KELVIN-HELMHOLTZ INSTABILITY

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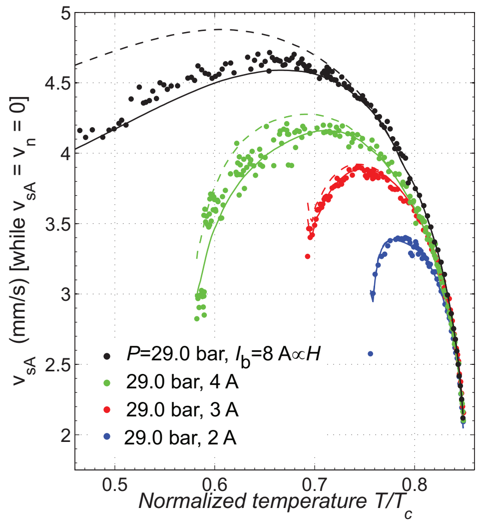
The Kelvin-Helmholtz instability (KH) is one of the celebrated instabilities of fluid flow, well-known in classical hydrodynamics since the 1860ies when it was discussed by William Thomson (1871) and Hermann von Helmholtz (1868) for two fluid layers of different density in relative shear flow. Spectacular manifestations are visible on the giant gas planets Saturn and Jupiter, at the interfaces separating different bands of rotating flow. For inviscid flow, the condition for the instability was shown to depend on the difference of the velocities parallel to the interface on either side of the boundary. Ever since the incorporation of viscous effects has proven problematic. In superfluids dissipative losses can be identified and accounted for more easily. Exact correspondence between experiment and theory then becomes tractable.

The historical instability condition on the relative velocity $\mid v_1 - v_2 \mid$ of the fluid layers (1) and (2) with densities ρ_1 and ρ_2 is given by

$$\frac{\rho_1 \rho_2}{\rho_1 + \rho_2}(v_1 - v_2)^2 = 2\sqrt{\sigma F}\ . \qquad (1)$$

The interface is characterized by its surface tension σ and its restoring force F. For instance, in the case of two layers in the ocean with different salinity or temperature $F = g \mid \rho_2 - \rho_1 \mid$ is the gravitational force. The interface becomes unstable when the free energy of the perturbed interface drops below that of the flat surface.



In superfluids the instability condition has to be recast in the form

$$\frac{1}{2}\rho_{s1}\left(v_{s1} - v_n\right)^2 + \frac{1}{2}\rho_{s2}\left(v_{s2} - v_n\right)^2 = \sqrt{\sigma F}\ . \qquad (2)$$

To preserve Galilean invariance, the two superfluid velocities v_{s1} and v_{s2} are now expressed with respect to the velocity v_n of the normal component (the thermal excitations).

The prime superfluid example is the instability at the interface separating the A and B phases of superfluid ^{3}He, discovered in 2002 [R. Blaauwgeers *et al.*, PRL **89**, 155301]. Here the restoring force arises from the difference in the magnetic energy of the two phases in the inhomogeneous externally applied magnetic field, needed to stabilize the AB interface: $F = \frac{1}{2}\left(\chi_A - \chi_B\right)\nabla H^2$. We have made extensive measurements of the instability as a function of temperature T, liquid pressure P, and magnetic field H. An example is shown in the above figure. A larger than expected correction from the magnetic field is needed to bring the curve according to Eq. (2) (*dashed-line curves*) down to fit the measured result (*solid-line curves*). The measured properties and their robustness provide further credence to the hypothesis that the KH instability could be the source for the famous glitches in the relaxation of neutron star rotation with time [A. Mastrano & A. Melatos, Mon. Not. R. Astron. Soc. **361**,927 (2005)].